BI-TP 2000/14
IRB-TH-5/00
hep-ph/0102xxx

Pinching phenomenon: Central feature in out of equilibrium thermal field theories

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Abstract

We study out of equilibrium thermal field theories with switching on the interaction occurring at finite time. We continue to study a formulation exploiting the concept of projected functions (PF) and Wigner transforms of projected functions (WTPF), for which convolution products between these functions can be achieved in a closed form without use of the gradient expansion. Many of the functions, appearing in the low orders of the perturbation expansion (bare propagators, one-loop self-energies, retarded and advanced components of the resummed propagator, ...) belong to the class of PF or WTPF. However, WTPF's are completely determined by their $X_0 \rightarrow +\infty$ limit and, thus, cannot be the carriers of relaxation phenomena. Furthermore, we observe that the functions capable of carrying relaxation phenomena (non-WTPF) emerge in the expressions containing mixed products (i.e., products of retarded and advanced propagators and self-energies; ill-defined in the usual formulation with the Keldysh time-path). In particular, to predict the time dependence of the system, one has to use equal-time Green functions (particle number, etc.). These are obtained by inverse Wigner transform (simple integration over energy in the case of equal time). The result of this operation is that all terms originating from WTPF will be constants in time (and equal to zero in most cases), and only the non-WTPF terms contribute to time variation. As these are generated in mixed products, the pinching phenomenon is being promoted from an obstacle to the central feature of out of equilibrium thermal field theories.

We analyze the pinching phenomenon in some detail. In the case of naive pinching (product only of retarded and advanced components of the bare propagator), for short times our calculation confirms the existence of the contributions linear in X_0. At very large times the contribution evolves to the usual pinching singularity. In Schwinger-Dyson equations the Keldysh component of self-energy always appears between the powers of retarded and advanced propagators. One easily finds that the mathematical expression

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corresponding to such a product is well defined even for multiple self-energy insertion contributions. We study the case of single self-energy insertion in more detail. We obtain the non-WTPF contribution which generates nontrivial X_0 dependence.

In the case of production of a photon from QCD plasma (finite-lifetime effect) approximate analytic results from our approach are almost identical to those obtained by S. -Y. Wang and D. Boyanovsky, who use the dynamical renormalization group approach.

PACS numbers: 05.70Ln, 11.10Wx, 11.15Bt, 122.38Mh, 12.38Cy

1 Introduction

Important aspects of modern physics depend very much on our understanding of nonequilibrium phenomena.

Many years of development of out of equilibrium thermal field theory (TFT) [1, 2] have resulted in slow but steady progress [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

For almost equilibrated systems, at infinite time after switching on the interaction, a number of results are valid at the lowest order in the gradient expansion [32, 33, 34]: the cancellation of collinear [16, 17] and pinching singularities [18, 19, 20, 21, 22, 23], the extension of the hard thermal loop (HTL) approximation [24, 25, 26] to out of equilibrium [27, 28], and applications to heavy-ion collisions [29, 30, 31].

For some problems, e.g., heavy-ion collisions, the above limitations are undesired. If one wants to consider large deviations from equilibrium, one should go beyond the gradient expansion. One cannot wait infinitely long as these systems go apart after a very short time, probably without reaching the stage of equilibrium. In nuclear collisions, short-time scale features have been studied in a number of papers [35, 36, 37, 38, 39, 40].

One of the characteristic features of out of equilibrium TFT is the appearance of mixed products of retarded and advanced propagators. In a formulation similar to equilibrium TFT, which use Keldysh time-path these terms have led to pinching singularities (or ill-defined δ^2 expressions). Many attempts to get rid of these can be classified as attempts within the zeroth order in the gradient expansion and as attempts to go beyond.

The first group of papers[18, 19, 16, 21, 23], although successful in eliminating pinching in some cases (in the single self-energy insertion contributions to the Keldysh component of the propagator in theories like QED and QCD [23], but not in multiple self-energy insertions), do not solve the problem of pinching in the theories like ϕ^3, ϕ^4, nor in the theory describing the $\rho - \pi$ interaction.

The second group of papers [20, 14, 41, 42, 43, 44, 45] use the finite switching-on time integration path (see Fig. 1). In these approaches, the seminal terms (the terms growing infinitely with X_0) appear instead of pinching .

In our recent paper [51] we have studied out of equilibrium thermal field theories with switching on the interaction occurring at finite time. We observe that many of the functions, appearing in the low orders of the perturbation expansion, belong to the class of projected functions (PF in further text) or Wigner transforms of projected functions (WTPF). These functions have particularly simple multiplication rules. However, WTPF are completely determined by their $X_0 \to +\infty$ limit and, thus, cannot be carriers of relaxation phenomena. Furthermore, we observe that the functions capable of carrying relaxation phenomena (non-WTPF) emerge in

the expressions containing mixed products (i.e., products of retarded and advanced propagators and self-energies). It is important to note that the non-WTPF contribution emerges even in the case when the time path is not "pinched" by two infinitely close poles; it is enough that the poles (or more complicated singularities) are situated on the opposite sides of the integration path. This means that the calculation with resummed propagators with complex poles (if we manage to have such!) will as well produce non-WTPF terms and generate relaxation.

In the present paper we develop these ideas further. In particular, to predict the time dependence of the system, one has to use equal-time Green functions. These are obtained by inverse Wigner transform (simple integration over energy in the case of equal time). The result of this operation is that all terms originating from WTPF are constants in time (and equal to zero in most cases), and only the non-WTPF terms contribute to time variation. As these are generated by mixed products, the pinching phenomenon is being promoted from an obstacle to the central feature of out-of-equilibrium thermal field theories.

In the present paper, after a short recapitulation of earlier results (Sec. II), we introduce (Sec. II.1) the function $sign(p_0, \omega_p)$, which is the generalization of the $sign(p_0)$ function. Especially in the case of particles with spin one use the adventages of the function $sign(p_0, \omega_p)$ to obtain the expressions "manifestly" retarded or advanced. In Sec. II.2 we establish the connection between two-point functions and equal-time functions (number operator, etc.), and prove that time-dependent contributions to equal-time functions come solely from non-WTPF.

In Sec. III we analyze pinching phenomenon in some detail. We further reduce the case of naive pinching (product of only a retarded and an advanced component), to the problem of pinching between two infinitely close poles. For short times, our calculation confirms the existence of the contributions linear in X_0. At very long times, the contribution evolves to the usual pinching singularity.

In Schwinger-Dyson equations (Sec.IV) the Keldysh component of self-energy always appears between the powers of retarded and advanced propagators . One easily finds that the mathematical expression corresponding to such a product is well defined even for multiple self-energy insertion contributions (Sec. IV.1). In the single self-energy insertion case (Sec. IV.2), one obtains two contributions. The one corresponding to pinching in the Keldysh time path approach, owing to the "ϵ"-shift becomes just the usual WTPF consisting of only one type of R/A components of the propagator and self-energy. The other contribution is non-FTPF; it generates nontrivial X_0 dependence.

In this paper it is important to preserve strict "ϵ" bookeeping the importance of which has been known since the discussion on the proper analytical continuation between the imaginary time formalism and the R/A approach [47, 48, 49, 50] in the real time formalism at equilibrium.

Our approach is an alternative to the dynamical renormalization group (DRG) approach [41, 42, 43]. Whereas we find the way to work with Feynman diagrams in the energy-momentum space and do not use the gradient expansion (at least in the low orders of the perturbation expansion), in the DRG approach one relies more on the differential equations with the gradient expansion as necessary tool. Nevertheless, the results (in our case, the results of the research in progress) are sometimes very similar. For a better understanding, one should compare the time dependence of specific processes calculated using both methods.

2 Out of equilibrium setup

In our previous paper [51] we have formulated the approach appropriate for the dynamical situation arising when the system starts its evolution at finite time (for simplicity, we take $t_i = 0$). In this formulation, the time integration follows the finite switching-on time path (see Fig. 1).

To understand the limitations coming from the finite switching-on time, we start with the two-point function $G(x,y)$. The quantities x and y are four-vector variables with time components in the range $0 < x_0, y_0 < \infty$. We define the Wigner variables s (relative space-time, relative variable) and X (average space-time, slow variable) as usual:

$$X = \frac{x+y}{2}, \;\; s = x - y,$$
$$G(x,y) = G(X + \frac{s}{2}, X - \frac{s}{2}). \tag{2.1}$$

The lower limit on x_0, y_0 implies conditions on X_0 and s_0: $0 < X_0, \;\; -2X_0 < s_0 < 2X_0$. The two-point function can be expressed in terms of the Wigner transform (i.e., the Fourier integral with respect to s_0, s_i):

$$G(X + \frac{s}{2}, X - \frac{s}{2}) = (2\pi)^{-4} \int d^4p e^{-i(p_0 s_0 - \vec{p}\vec{s})} G(p_0, \vec{p}; X). \tag{2.2}$$

Here

$$G(p_0, \vec{p}; X) = \int_{-2X_0}^{2X_0} ds_0 \int d^3s e^{i(p_0 s_0 - \vec{p}\vec{s})} G(X + \frac{s}{2}, X - \frac{s}{2}). \tag{2.3}$$

We have found that the low orders in the perturbation expansion are characterized by the appearance of very special two-point functions, we call them projected functions. Projected functions (truncated, "mutilated function" [46]) possess the following properties: the function does not change with $\vec{X}$ (homogeneity assumption), it is a function of $(s_0, \vec{s})$ within the interval $-2X_0 < s_0 < 2X_0$ and identical to zero outside the interval:

$$F(X + \frac{s}{2}, X - \frac{s}{2}) = \Theta(X_0)\Theta(2X_0 - s_0)\Theta(2X_0 + s_0)\bar{F}(s_0, \vec{s})$$
$$\bar{F}(s_0, \vec{s}) = \lim_{X_0 \to \infty} F(X + \frac{s}{2}, X - \frac{s}{2}). \tag{2.4}$$

The projected function can be viewed as the projection of the whole function $F_\infty(s_0, \vec{s}) = F(s_0, \vec{s})$ (i.e. the function defined at $X_0 = +\infty$ which uses the whole s_0 axis as a carrier) to its finite carrier. The projection operator is $P_{X_0}(s_0) = \Theta(X_0)\Theta(2X_0 - s_0)\Theta(2X_0 + s_0)$: $F_{X_0} = P_{X_0}(s_0) F_\infty$. The Wigner transform of the projected function (WTPF) at the given time X_0 may be obtained using the Wigner transform of the projected function (WTPF) at the time $X_0 = +\infty$.

$$F_\infty(p_0, \vec{p}) = \int_{-\infty}^{\infty} ds_0 \int d^3s e^{i(p_0 s_0 - \vec{p}\vec{s})} F(s_0, \vec{s}), \tag{2.5}$$

and the projection operator P_{X_0}

$$F_{X_0}(p_0,\vec{p}) = \int_{-\infty}^{\infty} dp_0' P_{X_0}(p_0,p_0') F_\infty(p_0',\vec{p}), \tag{2.6}$$

where

$$P_{X_0}(p_0,p_0') = \frac{1}{2\pi}\Theta(X_0)\int_{-2X_0}^{2X_0} ds_0 e^{is_0(p_0-p_0')} = \frac{1}{\pi}\Theta(X_0)\frac{\sin(2X_0(p_0-p_0'))}{p_0-p_0'} \tag{2.7}$$

is the Fourier transform of the projector and the inverse transform is given by

$$E^{-is_0p_0'}\Theta(X_0)\Theta(2X_0+s_0)\Theta(2X_0-s_0) = \int dp_0 e^{-is_0p_0} P_{X_0}(p_0,p_0'). \tag{2.8}$$

The assumption of the homogeneity in space coordinates excludes any dependence on $\vec{X}$ and we omit it as an argument of the function.

It is important to note that

$$\lim_{X_0\to\infty} P_{X_0}(p_0,p_0') = \lim_{X_0\to\infty}\frac{1}{\pi}\frac{\sin(2X_0(p_0-p_0'))}{p_0-p_0'} = \delta(p_0-p_0'), \tag{2.9}$$

and

$$\int dp_0 P_{X_0}(p_0,p_{01}) = \frac{1}{2i\pi}\int_{-\infty}^{\infty} dp_0\left(\frac{e^{2iX_0(p_o-p_{0,1})}-1}{p_o-p_{0,1}} - \frac{e^{-2iX_0(p_0-p_{0,1})}-1}{p_0-p_{0,1}}\right) = 1. \tag{2.10}$$

The last equality is obtained by closing the integration path in the first term from above and in the second term from below.

Evidently, there is a hierarchy of the P_{X_0} projectors:

$$P_{X_{0,M}}(p_0,p"_0) = \int dp_0' P_{X_0}(p_0,p_0') P_{X_0'}(p_0',p"_0), \quad X_{0,M} = min(X_0,X_0'). \tag{2.11}$$

For further analysis, the analytic properties of the WTPF in the $X_0\to\infty$ limit as a function of complex energy are very important. We define the following properties: (1) the function of p_0 is analytic above (below) the real axis, (2) the function goes to zero as $|p_0|$ approaches infinity in the upper (lower) semiplane. The choice above (below) and upper (lower) refers to R (A) components. It is easy to recognize that the properties (1) and (2) are just the definition of the retarded (advanced) function. However, it is nontrivial, and not always true, that the functions with the R (A) index satisfy them.

Important examples of projected functions satisfying assumptions (1) and (2) are poles in the energy plane, retarded, advanced, and Keldysh components of free propagators, and one-loop self-energies.

The Wigner transform of the convolution product of two-point functions

$$C = A * B \Leftrightarrow C(x,y) = \int dz A(x,z)B(z,y) \tag{2.12}$$

is given by the gradient expansion (note that we have assumed the homogeneity in space coordinates, which excludes any dependence on $\vec{X}$):

$$C_{X_0}(p_0,\vec{p}) = e^{-i\diamondsuit} A_{X_0}(p_0,\vec{p}) B_{X_0}(p_0,\vec{p}), \quad \diamondsuit = \frac{1}{2}(\partial^A_{X_0}\partial^B_{p_0}. - \partial^A_{p_0}\partial^B_{X_0}). \tag{2.13}$$

A much simpler expression is valid for A and B being projected functions:

$$C_{X_0}(p_0,\vec{p}) = \int dp_{01}dp_{02}P_{X_0}(p_0,\frac{p_{01}+p_{02}}{2})\frac{1}{2\pi}\frac{ie^{-iX_0(p_{01}-p_{02}+i\epsilon)}}{p_{01}-p_{02}+i\epsilon}A_\infty(p_{01},\vec{p})B_\infty(p_{02},\vec{p}). \quad (2.14)$$

Under the assumption that A or B satisfy (1) and (2) (A as advanced or B as retarded) Eq. (2.14) can be integrated even further. We obtain

$$C_{X_0}(p_0,\vec{p}) = \int dp_0' P_{X_0}(p_0,p_0')A_\infty(p_0',\vec{p})B_\infty(p_0',\vec{p}). \quad (2.15)$$

The convolution product of two two-point functions which are WTPF's and satisfy (1) and (2) is also a WTPF. This product is then expressed through the projection operator acting on a simple product of two WTPF's given at $X_0 = \infty$.

As expected, in the $X_0 = \infty$ limit, Eq.(2.15) becomes a simple product

$$\lim_{X_0\to\infty} C_{X_0}(p_0,\vec{p}) = A_\infty(p_0,\vec{p})B_\infty(p_0,\vec{p}). \quad (2.16)$$

At finite X_0, Eq.(2.15) exhibits a smearing of energy (as much as it is necessary to preserve the uncertainty relations).

The product of n two-point functions is obtained by repeating the above procedure:

$$C_{X_0}(p_0,\vec{p}) = \int \prod_{j=1}^{n-1}(dp_{0,j})dp_{0,n}P_{X_0}(p_0,(p_{0,1}+p_{0,n})/2)$$
$$\times \prod_{j=1}^{n-1}\left(A_{j,\infty}(p_{0,j},\vec{p})\frac{1}{2\pi}\frac{i}{p_{0,j}-p_{0,j+1}+i\epsilon}\right)e^{-iX_0(p_{0,1}-p_{0,n}+i(n-1)\epsilon)}A_{n,\infty}(p_{0,n},\vec{p}). \quad (2.17)$$

We note here: the condition that the intermediate products should also be projected functions requires that at least n-1 of the functions in the product should satisfy assumptions (1) and (2) (the retarded should be on the right-hand side and the advanced on the left-hand side, and the function that eventually does not satisfy (1) and (2) should be inbetween). However, this is not the order in which the components appear in the Schwinger-Dyson equation.

Then one can perform all integrations except one to obtain

$$C_{X_0}(p_0,\vec{p}) = \int dp_{0,1}P_{X_0}(p_0,p_{0,1})\prod_{j=1}^{n}A_{j,\infty}(p_{0,1},\vec{p}). \quad (2.18)$$

2.1 Function $sign(p_0,\omega_p)$, propagator and self-energy

For the retarded (advanced) and Keldysh components of the bare propagator one obtains (owing to the relation (2.6) it is enough to give their form at infinite time $X_0 = +\infty$):

$$G_{R(A),\infty}(p) = (-G_{1,1}+G_{1,2(2,1)})_\infty(p) = \frac{-i}{p^2-m^2\pm 2i\epsilon p_0}. \quad (2.19)$$

$$G_{K,\infty}(p) = (G_{1,1}+G_{2,2})_\infty(p) = 2\pi[1+2f(\omega_p)]\delta(p^2-m^2). \quad (2.20)$$

Our "$_\infty$" components coincide with the usual Keldysh-integration-path propagators. These expressions are easily generalized to the case of other spin and statistic assignment.

The Keldysh component as a function of p_0 does not satisfy assumptions (1) and (2), however, we can decompose it into the sum of functions which satisfy them either as retarded or as advanced functions. This trick we repeat below in the case of Σ_K. To do so we start with an identity:

$$\delta(x-y) = \frac{i}{2\pi}\gamma(\frac{x}{y})[\frac{1}{x-y+i\epsilon} - \frac{1}{x-y-i\epsilon}] + \mathcal{O}(\epsilon^2), \tag{2.21}$$

where $\gamma(1) = 1$, otherwise, $\gamma(x/y)$ is only weakly constrained: it should be analytic around $x/y = 1$. Equation (2.21) is used to generate the following identity:

$$\delta(p_0^2 - \omega_p^2) = \frac{i}{2\pi} sign(p_o, \omega_p)[\frac{1}{p_0^2 - \omega_p^2 + 2i\epsilon p_0} - \frac{1}{p_0^2 - \omega_p^2 - 2i\epsilon p_0}] + \mathcal{O}(\epsilon^2). \tag{2.22}$$

In the above identity we have substituted the usual $sign(p_0)$ function by a new (user friendly) function $sign(p_0, \omega_p)$, which we specify as an alternative between

$$sign(p_0, \omega_p) = sign(p_o), \frac{p_0}{\omega_p}, \frac{\omega_p}{p_0}, \left(\frac{p_0}{\omega_p}\right)^3, \left(\frac{\omega_p}{p_0}\right)^3, \ldots \tag{2.23}$$

Evidently, the function $sign(p_0, \omega_p)$ at $p_0 = \pm\omega_p$ for all offered possibilities reduces to $sign(p_0)$ and the identity is valid. The choice of the appropriate form of $sign(p_0, \omega_p)$ should guarantee that in the perturbative expansion integrals over p_0 converge (in such a way that two terms in Eq. (2.24), $G_{K,R}$ and $G_{K,A}$, could be treated separately) at $|p_0| = \infty$ and no additional singularities appear at finite p_0 (especially not at $|p_0| = 0$). This choice might be different for different terms. The difference between any two choices (when multiplied by $\delta(p_0^2 - \omega_p^2)$) is $\mathcal{O}(\epsilon^2)$. In the absence of pathology, this difference vanishes in the $\epsilon \to 0$ limit. The usual $sign(p_0)$ (first, not a recommended [23] choice), owing to its nonanalytic nature, has prevented the use of Cauchy integrals in the expressions containing $G_{K,R,\infty}$. The choice $\frac{p_0}{\omega_p}$ is a default choice, one uses it if the integrals converge. The choice $\frac{\omega_p}{p_0}$ is useful; with respect to the default choice, it reduces the power of p_0 by two units; if there is a factor p_0 in the integrand, this choice will not produce extra singularities at $p_0 = 0$. Similarly one can decide on the use of other choices. Having made proper choices, in the loop integration one can integrate over p_0 as first. This will result in manifestly retarded (advanced) functions. The ϵ parameter, which regulates these expressions, should be kept uniformly finite during the calculations, and the limit $\epsilon \to 0$ should be taken last of all [8]. This specially means that $\lim_{X_0\to\infty} \exp(-X_0\epsilon) = 0$ and the terms containing this factor vanish in the $X_0 \to \infty$ limit. Now we can write

$$G_{K,\infty}(p) = -G_{K,R,\infty}(p) + G_{K,A,\infty}(p),$$

$$G_{K,R(A),\infty}(p) = (1 + 2f(\omega_p))sign(p_0, \omega_p)G_{R(A),\infty}(p). \tag{2.24}$$

To discuss the amputated one-loop self-energy, we start with (spin and internal symmetry indices are suppressed)

$$\Sigma(x, y) = ig^2 S(x, y)D(x, y),$$

$$\Sigma_{R(A)}(p) = -(\Sigma_{1,1} + \Sigma_{1,2(2,1)})(p),$$

$$\Sigma_{R(A),\infty}(p) = \pm\frac{ig^2}{2}\int\frac{d^4k}{(2\pi)^4}[h(k_0,\omega_k) + h(p_0-k_0,\omega_{p-k})]D_{R(A),\infty}(k)S_{R(A),\infty}(p-k)F,$$

$$\Sigma_K(p) = (\Sigma_{11} + \Sigma_{22})(p) = -\Sigma_{K,R}(p) + \Sigma_{K,A}(p)$$

$$\Sigma_{K,R(A),\infty}(p) = \mp\frac{ig^2}{2}\int\frac{d^4k}{(2\pi)^4}[1 + h(k_0,\omega_k)h(p_0-k_0,\omega_{p-k})]$$

$$D_{R(A),\infty}(k)S_{R(A),\infty}(p-k)F, \qquad (2.25)$$

where D and S are bare scalar propagators, $h(k_0,\omega_k) = -sign(k_0,\omega_k)1+2f(\omega_p)]$ and the factor $F = F(k_0,|\vec{k}|,p_0,|\vec{p}|,\vec{k}\vec{p},...)$ includes the information about spin and internal degrees of freedom ($F = 1$ if all particles are scalars).

In the calculation of R, A, and K components of self-energy we use general expressions given by Eqs. (II.23)-(II.25) of Ref. [23]. For particles with spin it is the appropriate choice of $sign(k_0,\omega_k)$ that makes the integral over k_0 convergent term-by-term. Then the integrals over D_RS_A and D_AS_R vanish; we are thus left with the pure RR(AA) cotribution to the R(A) component.

General analytic properties of the expressions of the type (2.25) are well known: there are discontinuities (cuts) along the real axis (or, better to say, displaced from the real axis by $-i\epsilon$ ($+i\epsilon$) for the retarded (advanced) component), starting at thresholds for various real processes.

2.2 Equal-time two-point functions

To define single-particle observables one has to study reduction of two-point functions to equal time ($x_0 = y_0 = t$ or $X_0 = t$, $s_0 = 0$) [3]. These can be obtained by inverse Wigner transform as

$$G(t,0,\vec{p}) = \frac{1}{2\pi}\int dp_0 G_{X_0=t}(p_0,\vec{p}). \qquad (2.26)$$

As an example of equal-time two-point function one can study the number operator. To define it, we start with the Keldysh component of the propagator:

$$G_K(x,y) = G_{1,2}(x,y) + G_{2,1}(x,y) =< \phi(x)\phi(y) + \phi(y)\phi(x) >, \qquad (2.27)$$

$$G_K(X_0,s_0,\vec{p}) = \int d^3s e^{-i\vec{p}\vec{s}} G_K(X+\frac{s}{2}, X-\frac{s}{2}) = (2\pi)^{-1}\int dp_0 e^{ip_0s_0} G_{X_0,K}(p_0,\vec{p}). \qquad (2.28)$$

At $x_0 = y_0 = t$ (i.e., $s_0 = 0$; $X_0 = t$) one finds (under the usual assumption that $< aa >$ and $< a^+a^+ >$ terms vanish), the relation between the number operator and the Keldysh component of the propagator is

$$< 2N_{\vec{p}}(t) + 1 >= \omega_p G_K(t,0,\vec{p}) = \frac{\omega_p}{2\pi}\int dp_0 G_{t,K}(p_0,\vec{p}). \qquad (2.29)$$

Further single-particle observables are generated with the help of $< N_t >$. In the case of bare fields, one obtains as expected:

$$< 2N^0_{\vec{p}} + 1 >= \frac{\omega_p}{2\pi}\int dp_0 G^0_{X_0,K}(p) = \frac{\omega_p}{2\pi}\int dp_0 dp'_0 P_{X_0}(p_0,p'_0) G^0_K(p'_0,\vec{p})$$
$$= \frac{\omega_p}{2\pi}\int dp'_0 G^0_K(p'_0,\vec{p}) = -Im(\int dp'_0 \frac{p'_0}{\pi}\frac{1+2f(\omega_p)}{p'^2_0 - \omega_p^2 + 2i\epsilon p'_0}),$$
$$= 1 + 2f(\omega_p). \tag{2.30}$$

The time independence of the right-hand side of relation (2.30) is a special case of a more general feature.

An equal-time two-point function coming from a retarded WTPF may be obtained with the help of Eqs. (2.26), (2.6), and(2.10) as (to avoid problems with Θ's, we understand here that setting $s_0 = 0$ is achieved by the limiting procedure $\lim_{s_0\to+0}$):

$$G_R(t,0,\vec{p}) = \frac{1}{2\pi}\int dp_0 G_{X_0,R}(p_0,\vec{p})$$
$$= \frac{1}{2\pi}\int dp_0 \int dp_{01} P_{X_0}(p_0,p_{01}) G_{\infty,R}(p_{01},\vec{p})$$
$$= \frac{1}{2\pi}\int dp_{01} G_{\infty,R}(p_{01},\vec{p}) = const(\vec{p}). \tag{2.31}$$

The integral over the WTPF $G_{X_0,R}$ does not change with time as the right-hand side refers to ∞ and not to X_0.It is even vanishing for expressions containing two or more bare retarded propagators in the product, as one can easily see by closing the path of integration over $p_{0,1}$ in Eq. (2.31) from above. One obtains the same result for the advanced function by closing the integration path from below.

This is a very important result, but is by no means surprising: the projected function is completely determined by its form at $X_0 = +\infty$ If it were to describe irreversible processes, it would violate causality. Now we may conclude that one really needs non-WTPFs to describe the time dependence of single-particle observables. These will emerge as a by-product of pinching.

3 Examples of pinching

3.1 Naive pinching with retarded and advanced propagators

The naive pinching singularity is represented by (at $X_0 = \infty$)

$$G_{pinch} = G_R * G_A, \tag{3.1}$$

where $G_{R(A)}$ is given by (2.19). One can decompose $G_{R(A),\infty}$ into the sum of two poles

$$G_{R(A),\infty}(p) = \frac{-i}{2\omega_p}\left(\frac{1}{p_0 \pm i\epsilon - \omega_p} - \frac{1}{p_0 \pm i\epsilon + \omega_p}\right), \tag{3.2}$$

so it is enough to study pinching between two infinitely close poles.

3.2 Pinching between two infinitely close poles

We assume the contribution of the pole infinitely close to the real axis:

$$\mathcal{G}_{R(A),\infty,pole}(p_0) = \frac{1}{p_0 - \bar{p}_0 \pm i\mu}, \tag{3.3}$$

where $\bar{p}_0$ is real and $\mu > \epsilon/2$. It satisfies assumptions (1) and (2) with the $+$ sign as retarded function, and with the $-$ sign as advanced functions.

The product $G_{R,pole} * G_{A,pole}$ is easily obtained by substituting $G_{R(A),\infty,pole}$ into (2.14). We choose new variables $P_0 = (p_{01} + p_{02})/2$ and $\Delta_0 = p_{01} - p_{02}$, and integrate over Δ_0 (care is necessary as ϵ and μ are both infinitely small quantities) to obtain

$$\begin{aligned} C_{X_0}(p_0) &= \int dP_0 P_{X_0}(p_0, P_0) \frac{1}{P_0 - \bar{p}_0 + i\mu - i\epsilon/2} \frac{1}{P_0 - \bar{p}_0 - i\mu + i\epsilon/2} \\ &+ \int dP_0 P_{X_0}(p_0, P_0) \frac{1}{\bar{p}_0 - p_0} \left(\frac{e^{2iX_0(P_0 - \bar{p}_0 + i\mu - i\epsilon/2)}}{2(P_0 - \bar{p}_0 + i\mu) - i\epsilon} + \frac{e^{-2iX_0(P_0 - \bar{p}_0 - i\mu + i\epsilon/2)}}{2(P_0 - \bar{p}_0 - i\mu) + i\epsilon} \right). \end{aligned} \tag{3.4}$$

The first term is the projected function; in the $X_0 \to \infty$, limit it becomes a usual example of pinching. The second term consists of two non-WTPF pieces.

Further integration gives (after introducing $\rho = \mu - \epsilon/2$)

$$C_{X_0}(p_0) = \frac{1 - e^{-2X_0\rho} \cos 2X_0(p_0 - \bar{p}_0)}{(p_0 - \bar{p}_0 + i\rho)(p_0 - \bar{p}_0 - i\rho)} + \frac{\rho e^{-2X_0\rho} \sin 2X_0(p_0 - \bar{p}_0)}{(p_0 - \bar{p}_0 + i\rho)(p_0 - \bar{p}_0 - i\rho)(p_0 - \bar{p}_0)}. \tag{3.5}$$

Expression (3.5) can be studied at different times. At very very large, but finite time (i.e., such that $\kappa/\rho \exp(-2X_0\rho) << 1$, where κ is a typical energy scale of the problem) one can ignore the $exp(-2X_0\rho)$ terms and obtains the usual Keldysh-path pinching. Needless to say, as ρ is arbitrarily small, the time should be "arbitrarily very very" large.

Ignoring the intermediate scales, we come to the finite-time scale. At this scale $X_0\rho <<$ 1 and the exponential can be substituted by "1". For large times ($X_0 >> \kappa^{-1}$), one can approximate [43]

$$\begin{aligned} &\frac{\sin 2X_0(p_0 - \bar{p}_0)}{p_0 - \bar{p}_0} \approx \pi\delta(p_0 - \bar{p}_0), \\ &\frac{\rho \sin 2X_0(p_0 - \bar{p}_0)}{(p_0 - \bar{p}_0 + i\rho)(p_0 - \bar{p}_0 - i\rho)(p_0 - \bar{p}_0)} \approx 2\pi X_0 \delta(p_0 - \bar{p}_0), \\ &\int_{-\infty}^{\infty} dp_0 f(p_0) \frac{\sin^2 X_0(p_0 - \bar{p}_0)}{(p_0 - \bar{p}_0)^2} \approx \pi X_0 f(\bar{p}_0) + \frac{\mathcal{P}}{2} \int_{-\infty}^{\infty} dp_0 \frac{f(p_0) - f(\bar{p}_0)}{(p_0 - \bar{p}_0)^2}. \end{aligned} \tag{3.6}$$

Finally, one obtains

$$\int C_{X_0}(p_0) f(p_0) dp_0 \approx 4\pi X_0 f(\bar{p}_0) + \int dp_0 \mathcal{P} \frac{f(p_0) - f(\bar{p}_0)}{(p_0 - \bar{p}_0)^2}. \tag{3.7}$$

In this expression there is a term proportional to $X_0\delta(p_0 - \bar{p}_0)$ (seminal term according to some authors).

As expected, naive pinching at finite times gives contributions proportional to X_0; at "very very large" times it develops usual pinching singularities.

4 Elimination of pinching in Schwinger-Dyson equations

We write the Schwinger-Dyson equations in the form

$$\mathcal{G}_R = G_R + iG_R * \Sigma_R * \mathcal{G}_R, \quad \mathcal{G}_A = G_A + iG_A * \Sigma_A * \mathcal{G}_A,$$

$$\mathcal{G}_K = iG_R * \Sigma_K * \mathcal{G}_A + iG_K * \Sigma_A * \mathcal{G}_A + iG_R * \Sigma_R * \mathcal{G}_K. \tag{4.1}$$

We can expand Eqs. (4.1) to obtain ($\Sigma_K = -\Sigma_{K,R} + \Sigma_{K,A}$):

$$\mathcal{G}_R = \sum_{n=0}^{\infty} (G_R * i\Sigma_R *)^n G_R,$$

$$\mathcal{G}_K = \sum_{n=0}^{\infty} G_{K,n},$$

$$G_{K,n} = -(G_R * i\Sigma_R *)^n h G_R + h G_A (* i\Sigma_A * G_A)^n$$

$$+ \sum_{p=0}^{n-1} G_R (i\Sigma_R * G_R)^p * (-\bar{\Sigma}_{K,R} + \bar{\Sigma}_{K,A})(i\Sigma_A * G_A)^{n-p-1}, \tag{4.2}$$

where $\bar{\Sigma}_{K,R(A)} = h\Sigma_{R(A)} + \Sigma_{K,R(A)}$. Equations (4.2) are the forms in which pinching appears in out of equilibrium thermal field theories.

In fact, expression (4.2) is, term by term, free of pinching. To see this, one chooses a typical term containing $\bar{\Sigma}_{K,R}$ (the terms containing $\bar{\Sigma}_{K,A}$ are then obtainable by complex conjugation). For fixed n and $m = n - p - 1$, one can perform all integrations between either RR or AA factors (note the bookkeeping of ϵ's)

$$\mathcal{G}_{K,R,n,m} = (G_R * i\Sigma_R *)^n G_R * (-i\bar{\Sigma}_{K,R}) * (G_A * i\Sigma_A *)^m G_A,$$

$$\mathcal{G}_{X_0,K,R,n,m} = \int dp_{0,1} dq_{0,1} P_{X_0}(p_0, \frac{p_{0,1} + q_{0,1}}{2})$$

$$\prod_{j=0}^{n-1} \left(G_{\infty,R}(p_{0,1} + i2j\epsilon) i\Sigma_{\infty,R}(p_{0,1} + i(2j+1)\epsilon) \right) G_{\infty,R}(p_{0,1} + i2n\epsilon)$$

$$(-i\bar{\Sigma}_{\infty,K,R}(p_{0,1} + i(2n+1)\epsilon)) \frac{i}{2\pi} \frac{e^{-iX_0(p_{01} - q_{01} + i2(m+n+1)\epsilon)}}{p_{01} - q_{01} + i2(m+n+1)\epsilon} G_{\infty,A}(q_{0,1} - i2m\epsilon)$$

$$\prod_{l=0}^{m-1} \left(i\Sigma_{\infty,A}(q_{0,1} - i(2m-2l-1)\epsilon) G_{\infty,A}(q_{0,1} - i2(m-l-1)\epsilon) \right). \tag{4.3}$$

Owing to the poles of G_R and the cuts of Σ_R and $\Sigma_{K,R}$ below the real axis, and the divergence of the $e^{-iX_0 p_{0,1}}$ factor when $Im p_{0,1} \to +\infty$, the integral over $p_{0,1}$ (for similar reasons, also the integral over $q_{0,1}$) cannot be evaluated analytically. To find the analytical properties of $\mathcal{G}_{K,R,n,m}$, we have to study integrals of the type

$$\mathcal{I}_{X_0}(p_{0,1}, \epsilon, r) = \int_{-\infty}^{+\infty} dq_{0,1} \frac{e^{iX_0 q_{0,1}}}{p_{0,1} - q_{0i,1} + ir\epsilon} F_{\infty,A}(q_{0,1}), \tag{4.4}$$

where $r > 1$. The function F possesses the singularities only within the strip $\epsilon < Imq_{0,1} < r\epsilon$ and vanishes when $|q_{0,1}| \to \infty$ outside the strip.Thus it satisfies assumptions (1) and (2) in the lower semiplane.It is easy to see that for $Imp_{0,1} > 0$ or $Imp_{0,1} < -c-r\epsilon$, where $c > 0$ is a small finite number, the integration path can be shifted down away from the strip with singularities. The integration over $q_{0,1}$ is regular even in the $\epsilon \to 0$ limit, and $\mathcal{I}(p_{0,1})$ represents the function analytical in $p_{0,1}$ for $p_{0,1}$ outside the strip $-c - r\epsilon < Imp_{0,1} < 0$ and satisfying $|Imp_{0,1}| < \infty$.

Also in the integration over $p_{0,1}$ along the real axis, all "nearby" singularities are confined within the strip below the real axis and one can move the integration path for the $p_{0,1}$ integration uphill to obtain regular integrals. Thus there is no pinching in the $p_{0,1}$ integration.

Owing to the fact that the factor $P_{X_0}(p_0, \frac{p_{0,1}+q_{0,1}}{2}) = \frac{1}{i\pi}\frac{e^{iX_0(2p_0-p_{0,1}-q_{0,1})}-e^{-iX_0(2p_0-p_{0,1}-q_{0,1})}}{2p_0-p_{0,1}-q_{0,1}}$ is regular at $2p_0 - p_{0,1} - q_{0,1} = 0$ and for all p_0, $p_{0,1}$, and $q_{0,1}$ satisfying $|Imp_0|$,$|Imp_{0,1}|$, and $|Imq_{0,1}| < \infty$, its presence in (4.3) will not change our conclusion that $\mathcal{G}_{X_0,R,n,m}$ as represented in (4.3) is free from pinching.

Thus we have shown that pinching is absent from the contributions to $\mathcal{G}_K$ with an arbitrary number of self-energy insertions.

In the single self-energy insertion approximation, one can perform the proof in more detail.

4.1 Elimination of pinching in the single-self-energy insertion approximation

The single-self-energy-insertion approximation to the Keldysh component of the propagator is expressed as [23] (we treat only the scalar case, superscript "0" bare, superscript "1" one-loop contribution)

$$G_K = G^1_{Kp,R} + G^1_{Kp,A} + G^0_{Kr} + G^1_{Kr} + ...,$$

$$G^1_{Kp,R} = -iG_R * \bar{\Sigma}_{K,R} * G_A, \quad G^1_{Kp,A} = iG_R * \bar{\Sigma}_{K,A} * G_A,$$

$$G^0_{Kr} + G^1_{Kr} = h(G_R - G_A) + iG_R * h\Sigma_R * G_R$$

$$-iG_A * h\Sigma_A * G_A. \tag{4.5}$$

In expression (4.5), $G^1_{Kp,R}$ and $G^1_{Kp,A}$ are potentially ill-defined, while G^0_{Kr} and G^1_{Kr} are explicitly free from pinching.

To see what happens in full detail, we start with the contribution containing $\bar{\Sigma}_{K,R}$ (we do not indicate explicitly the dependence on $\vec{p}$ on the right-hand sides of the following equations):

$$G^1_{Kp,R} = -iG_R * \bar{\Sigma}_{K,R} * G_A,$$

$$G^1_{X_0,Kp,R}(p_o,\vec{p}) = -i\int dp_{01}dp_{02}dp_{03}P_{X_0}(p_0, \frac{p_{01}+p_{03}}{2})G_R(p_{01})\frac{i}{2\pi}\frac{e^{-iX_0(p_{01}-p_{02}+i\epsilon)}}{p_{01}-p_{02}+i\epsilon}$$

$$\bar{\Sigma}_{\infty,K,R}(p_{02})\frac{i}{2\pi}\frac{e^{-iX_0(p_{02}-p_{03}+i\epsilon)}}{p_{02}-p_{03}+i\epsilon}G_A(p_{03}), \tag{4.6}$$

Here we can integrate over p_{02} by closing the integration path from above. The only singularity closed in the integration path is situated at $p_{01}+i\epsilon$. The result is (note the care for ϵ's):

$$G^1_{X_0,Kp,R}(p_0,\vec{p}) = -i\int dp_{01}dp_{03}P_{X_0}(p_0,\frac{p_{01}+p_{03}}{2})G_R(p_{01})$$
$$\bar{\Sigma}_{\infty,K,R}(p_{01}+i\epsilon)\frac{i}{2\pi}\frac{e^{-iX_0(p_{01}-p_{03}+2i\epsilon)}}{p_{01}-p_{03}+2i\epsilon}G_A(p_{03}), \tag{4.7}$$

Now one can integrate over p_{03} by closing the integration path from above. The singularities closed within the path are situated at $p_{01}+2i\epsilon$ and at $\pm\omega_p+i\epsilon$.

$$G^1_{X_0,Kp,R}(p_0,\vec{p}) = -i\int dp_{01}P_{X_0}(p_0,p_{01})G_R(p_{01})\bar{\Sigma}_{\infty,K,R}(p_{01}+i\epsilon)G_A(p_{01}+2i\epsilon)$$
$$+\frac{1}{2\omega_p}\int dp_{01}G_R(p_{01})\bar{\Sigma}_{\infty,K,R}(p_{01}+i\epsilon)\sum_{\lambda=-1}^{1}\lambda P_{X_0}(p_0,\frac{p_{01}+\lambda\omega_p}{2})\frac{e^{-iX_0(p_{01}-\lambda\omega_p+i\epsilon)}}{p_{01}-\lambda\omega_p+i\epsilon}. \tag{4.8}$$

By inspecting the definitions of G_R and G_A in (2.19), one observes that $G_A(p_{01}+2i\epsilon) = G_R(p_{01})$, so that all functions appearing in (4.8) are retarded. There is no pinching, but we have obtained functions depending directly on time X_0, i.e., non-WTPF functions, which one cannot convolute further in an elegant way we have used here.

One can do the same with the term containing $\bar{\Sigma}_{K,A}$, but now one has to integrate over p_{02} by closing the path from below, and over p_{01} again closing path from below, the result is (now one needs $G_R(p_{03}-2i\epsilon) = G_A(p_{03})$)

$$G^1_{X_0,Kp,A}(p_0,\vec{p}) = i\int dp_{03}P_{X_0}(p_0,p_{03})G_R(p_{03}-2i\epsilon)\bar{\Sigma}_{\infty,K,A}(p_{01}+i\epsilon)G_A(p_{01})$$
$$-\frac{1}{2\omega_p}\int dp_{03}\sum_{\lambda=-1}^{1}\lambda P_{X_0}(p_0,\frac{p_{03}+\lambda\omega_p}{2})\frac{e^{iX_0(p_{03}-\lambda\omega_p-i\epsilon)}}{p_{03}-\lambda\omega_p-i\epsilon}\bar{\Sigma}_{\infty,K,A}(p_{03}-i\epsilon)G_A(p_{03}). \tag{4.9}$$

Now we add G_{Kr} to (4.8) and (4.9) and obtain (we can ignore "the surplus of ϵ" in $\bar{\Sigma}_{K,R(A)}$)

$$G^1_{X_0,K}(p_0,\vec{p}) = 2Im\Bigg(\int dp_{01}P_{X_0}(p_0,p_{01})G_R(p_{01},\vec{p})\Sigma_{\infty,K,R}(p_{01},\vec{p})G_R(p_{01},\vec{p})$$
$$+i\frac{1}{2\omega_p}\int dp_{01}G_R(p_{01},\vec{p})\bar{\Sigma}_{\infty,K,R}(p_{01},\vec{p})$$
$$\sum_{\lambda=-1}^{1}\lambda P_{X_0}(p_0,\frac{p_{01}+\lambda\omega_p}{2})\frac{e^{-iX_0(p_{01}-\lambda\omega_p-i\epsilon)}}{p_{01}-\lambda\omega_p+i\epsilon}\Bigg). \tag{4.10}$$

This expression is a function of two variables, p_0 and $\vec{p}$. It is the generalization of the usual mass shell condition. For fixed $\vec{p}$, it offers information about the shape of the distribution of particles as a function of time. If we are not interested in the shape of the distribution, we can integrate over p_0. The result is a one-loop contribution to the number operator. It tells us about the time dependence of the occupancy of a given set of particle states characterized by fixed $\vec{p}$. As the first term in Eq. (4.10) is a retarded function, it vanishes after integration. Thus the projected

function does not change the integrated distribution function! It only redistributes the given contribution within the shape. The function is symmetric under the change $p_0 \to -p_0$; thus division by 2 is equivalent to the projection to positive frequencies.

Yhe second term can be rearranged to obtain ($X_0 = t$)

$$
\begin{aligned}
&< 2N_{\vec{p}}(t) + 1 >=< 2N^0_{\vec{p}}(t) + 1 > + < 2N^1_{\vec{p}}(t) > +... \\
&= \frac{\omega}{2\pi}\int dp_0 G^0_{X_0,K}(p_0,\vec{p}) + \frac{\omega}{2\pi}\int dp_0 G^1_{X_0,K}(p_0,\vec{p}) \\
&= 1 + 2f(\omega_p) + \frac{\omega}{\pi} Im\left(\int dp_{01} G_R(p_{01})\bar{\Sigma}_{\infty,K,R}(p_{01}) G_R(p_{01})\right. \\
&\left.[1 - e^{-iX_0(p_{01}+i\epsilon)}(\cos X_0\omega_p + i\frac{p_{0,1}}{\omega_p}\sin X_0\omega_p)]\right). \qquad (4.11)
\end{aligned}
$$

The term proportional to 1 in braces is added for convenience; it vanishes upon integration over p_{01} in the upper hemisphere. The fact that WTPF do not contribute to Eq. (4.11) throws a new light on our approach: pinchlike contributions (i.e., those containing convolution products of both retarded and advanced components) are necessary to obtain nontrivial time dependence. As this fact will reappear in other expressions (even the calculation of retarded and advanced components from two-loop or more complicated Feynman diagrams) we may conclude that, indeed, pinchlike expressions represent "the body of evidence" that very important information is left "ill-defined" in the formulation using the Keldysh time path.

To understand the meaning of (4.11) we have to compare it with Eq. (10) from [44] (see also [45]) for "enhanced photon production from quark-gluon plasma - finite-lifetime effect":

$$
< N^1_{\vec{p}}(t) >=< N_{\vec{p}}(0) > + \frac{2}{\pi(2\pi)^3}\int_{-\infty}^{\infty} dp_{01} R(p_{01}) \frac{1-\cos[(p_{01}-\omega_p)t]}{\pi(p_{01}-\omega_p)^2}. \qquad (4.12)
$$

The differences are: 1) In this paper we treat only the scalar case. To adopt it for vector photons and spinor quarks, we have to substitute $\Sigma(p_0) \to 2\Sigma^T(p_0)$ (T for projection of Σ to its transverse part, factor 2 for two transverse degrees of freedom). 2) Wang and Boyanovsky use $R(p_0) = -Im\bar{\Sigma}_<(p_0)$, while we prefer the Keldysh component $\bar{\Sigma}_{K,R(A)}(p_0)$. The difference is a WTPF, so it does not contribute. However, there is the term with $Re\bar{\Sigma}_{K,R(A)}(p_0)$, which is not present in (4.12). It gives rise to extra oscilations. 3) We have used $-\omega_p^2 G_R(p_o,\vec{p})^2$, while in (4.12) one finds $(p_{01}-\omega_p)^{-2}$. Owing to this difference their integrand is no longer symmetric under the change $p_{01} \to -p_{01}$. This will be more important for contributions from larger $|p_{01}-\omega_p|$. This difference seems to come from the approximations intrinsic to the dynamical renormalisation group approach. 4) Nevertheless an approximate analytical calculation using HTL self-energies as input to photon-quark-antiquark analog (4.11) and to (4.12) [44] gives almost identical results.

5 Conclusions

We have studied out of equilibrium thermal field theories with switching on the interaction occurring at finite time. We have continued to study formulation exploiting the concept of

projected function (PF) and Wigner transform of projected function (WTPF),for which convolution products between these functions can be achieved in a closed form without use of the gradient expansion. Many of the functions, appearing in the low orders of the perturbation expansion (bare propagators, one-loop self-energies, retarded and advanced components of resummed propagator, ...) belong to the class of PF or WTPF. However, WTPF's are completely determined by their $X_0 \to +\infty$ limit and, thus, cannot be the carriers of relaxation phenomena. Furthermore, we have observed that the functions capable of carrying relaxation phenomena (non-WTPF) emerge in the expressions containing mixed products (i.e., the products of retarded and advanced propagators and self-energies; ill-defined in the usual formulation with the Keldysh time-path). In particular, to predict the time dependence of the system, one has to use equal-time Green functions (particle number, etc.). These are obtained by inverse Wigner transform (simple integration over energy in the case of equal time). The result of this operation is that all terms originating from WTPF's will be constants in time (and equal to zero in most cases), and only non-WTPF terms contribute to time variation. As these are generated in mixed products, the pinching phenomenon is being promoted from an obstacle to the central feature of out of equilibrium thermal field theories.

We have analyzed pinching phenomenon in some details. A general feature here is that in the expressions containing pinching in the Keldysh time-path formulation, simple products of retarded and advanced components become double integrals of corresponding quantities.

In the case of naive pinching (product only of retarded and andvanced component), at short times, our calculation confirms the existence of contributions linear in X_0. At very large times the contribution evolves to the usual pinching singularity. In this case pinching singularity appears as an artifact of the limiting procedure $X_0 \to \infty$.

In Schwinger-Dyson equations the Keldysh component of self-energy always appears between the powers of retarded and advanced propagators. One easily finds that the mathematical expression corresponding to such a product is well defined even for multiple self-energy insertion contributions. We have studied single self-energy insertion in more detail. We have obtained non-WTPF contribution that generates nontrivial X_0 dependence.

In the case of production of photons from QCD plasma (finite-lifetime effect), approximate analytic results from our approach are almost identical to those obtained by S. -Y. Wang and D. Boyanovsky, who use the dynamical renormalization group approach.

We may conclude that, indeed, out of equilibrium TFT, using the finite-time path and the recognition of basic quantities as WTPF's, retain all good properties of the Keldysh-time-path formulation (energy-momentum space description, Feynman diagrams), while removing the problem of illdefined quantities.

Acknowledgments I am especially indebted to Rolf Baier for many stimulating discussions. I also acknowledge financial support from the A. von Humboldt foundation. This work was supported by the Ministry of Science and Technology of the Republic of Croatia under contract No. 00980102.

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Figure Captions
Fig. 1: Finite switching-on time path.